No Time for the Hamiltonian Constraint

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The theory of loop quantum gravity (LQG) is one of the leading contenders for a theory of gravity at the Planck scale, and yet like all such contenders – string theory, causal set theory – LQG is fraught with a variety of difficulties. Most of these difficulties are technical in nature and are not burdened by the conceptual angst inherent in what has come to be known as "the problem of time." According to this problem, time is described by LQG as being "frozen" or missing from the world. In the following, I will address the problem of time by highlighting a tension between it and different interpretations of 'spacetime' and spacetime's relation to the mathematical manifold $\mathcal{M}$. I will use this tension and subsequent analysis to argue that the problem of time results not from the Hamiltonian constraint, as is often claimed, but due to our interpretation of LQG.

This paper is comprised as follows:

1 Primer on LQG

I will assume that my reader has some familiarity with the theory of LQG and will do my best to ignore much, though not all, of its technical machinery.[1] In this section, I will provide an overview of the mathematics required for a first order understanding of the problem of time and its relationship to interpretations of LQG.

The theory of LQG is a theory of "canonically quantized" general relativity (GR) formulated using a special set of "loop" variables. The canonical program quantizes physical fields using the method developed by Dirac. Before running GR through Dirac's procedure, in LQG we first rewrite the metric field g on the manifold $\mathcal{M}$ in terms of a pair of "tetrad" fields on $\mathcal{M}$. What mathematical form these fields take is not important, but what is important is that these fields are themselves defined by an $\mathfrak{su}(2)$-gauge field $\mathcal{A}$. Though it is the gauge variable $\mathcal{A}$ which gets canonically quantized, since the metric field is written in terms of it, I will often talk about quantizing the metric field g or the gravitational field represented by it. Once we have rewritten GR in terms of Ashtekar's new variables $(\mathcal{A}, \tilde{E})$, we quantize the gravitational field by running the Hamiltonian version of GR through Dirac's procedure. The first half of this process results in a classical theory but one in which the physics of GR has been squeezed into three constraints.[2] These constraints are restrictions on what trajectories, in the phase space of GR, count as being physical.

In order to quantize the theory, we upgrade the constraints by turning certain functions of the canonical variables into operators and requiring that the newly christened operator-constraints annihilate what will be the physical states of theory. In other words, schematically, before quantization we have three constraints of the form:

$$C_i(A_j, \tilde{E}_j) = 0 \qquad (i = 1, 2, 3). \tag{1}$$

Where A_j and $\tilde{E}_j$ play the role of our canonical position and momentum variables. As part of the quantization process, these constraints become:

$$\hat{C}_i(A_j, \tilde{E}_j)\Psi = 0. \tag{2}$$

These constraints are known as the Gauss, vector, and scalar constraints; though, the Gauss constraint is often referred to as the gauge constraint, the vector as the diffeomorphism constraint, and the scalar as the Hamiltonian constraint. The Gauss constraint requires the physical system of LQG to be invariant under an internal gauge transformation, the vector constraint requires the system to be invariant under spatial diffeomorphisms, and the scalar constraint requires the system to be invariant under a reparameterization of the

[1] For an introduction to the theory see (Gambini and Pullin, 2011) or (Norton, Quantum Ontology: spin-networks and spacetime, in preparation) and for a mathematically involved account of the theory see (Rovelli, 2004) or (Thiemann 2007). The general and well known results of LQG, reproduced below, can be be found in these sources.

[2] There are three constraints in the context of Ashtekar's reformulation and two constraints when written in terms of the original "lapse" and "shift functions."

time coordinate (Gambini and Pullin 2011, p.93-94, Rovelli 2004, p.146, 225). This current work is part of an industry debating whether or not these constraints require or suggest that variation across space and through time is either frozen or missing, but more on this to come.[3]

In addition to the constraint-operators, $\hat{C}_i(A_j, \tilde{E}_j)$, what have come to be known as the "area" and "volume operators" have been constructed, and each of these operators has been found to have a countable spectrum with a lower bound. According to the standard interpretation of these operators, physical regions and surfaces do not come in just any size. The area of physical surfaces and the volume of physical regions come in discrete "Planck-sized" units. How LQG predicates geometric size to physical space will not be important for the purposes of this paper; what will be important is that, according to LQG, the geometric size of physical regions and surfaces is limited to elements of discrete spectra.[4]

It is important to keep clear the difference between 'region' and 'surface' on the one hand and 'volume' and 'area' on the other. Though colloquially we often use 'volume' and 'region' interchangeably, we will not do so in the context of LQG (or GR for that matter). Allow me to distinguish these concepts. In the context of a mathematical manifold Σ, a surface is a two dimensional set of points, and a region is a three dimensional set. The area and volume of these sets are measures of their geometric size. If Σ is Euclidean, the geometric size of some rectangular surface is the product of its two perpendicular edges, and we call this geometric measure "area." This same distinction holds when considering *physical* regions and surfaces: area and volume are measures of the physical-geometric size of regions and surfaces. What physical regions and surfaces are depends on one's metaphysics of spacetime. According to a common form of substantivalism, physical regions and surfaces are collections of substantival spacetime points. Whereas, central to relationism is the view that physical regions and surfaces are codifications of spatial relations which hold between physical objects. What is new and beautiful about the theories of GR and LQG is that fixed regions and surfaces have variable geometric size depending on the mass and energy associated with the regions and surfaces.[5] For instance, in GR, given a single physically substantial surface (a set of substantival points), the size of the geometry, given by the

[3] It turns out that some version of the problem of time is found in any theory which utilizes the Hamiltonian version of GR. In other words, the problem of time is not a special problem for LQG (Earman 2002). For more on the problem of time see Isham (1991, 1992), Kuchař (1992), Earman (2002), and Wüthrich (2014).

[4] For more details on how LQG predicates size to physical areas and regions and why these sizes are elements of a discrete spectra see (Rovelli, 2004) or (Norton, in preparation).

[5] However, I doubt that this is actually true on a relationist account of spacetime. For instance, according to GR, in order to change the geometry we need to change the mass-energy distribution, which in turn describes a different collection of spatial relations between physical objects. While the area and volume do evolve, so too do relationally defined regions and relationally defined surfaces. Thus, it is not the case that one fixed region can have many volumes: as the volumes evolve so too do the relationally defined regions.

metric, and associated with the surface, will vary with the distribution of mass and energy. What is new in LQG and mentioned above is that the geometric size of physical surfaces and regions cannot get smaller and smaller. LQG predicts a lower bound for the geometric size of physical surfaces and regions.

In the following, I will present the standard argument from the Hamiltonian constraint for the absence of time in LQG. It is standardly claimed that due to the strange dynamics, or rather lack thereof, predicted by the Hamiltonian constraint, there is fundamentally no time in LQG. I will argue that this is the wrong diagnosis, and that time "goes missing" independent of the Hamiltonian constraint.

2 The problem of time

In presenting the problem of time, I will follow DeWitt's original 1967 account of the problem. Two things to note in this regard: firstly, Dewitt derives the problem of time in the context of "canonical quantum gravity" (CQG). CQG is formally distinct from LQG in so far as it does not rewrite the metric field g in terms of Ashtekar's gauge potential $\mathcal{A}$. In accordance with this choice of variables, the state functionals in canonical quantum gravity are $\Psi(\gamma_{ij})$ rather than $\Psi(A)$, where γ_{ij} is the three-dimensional projection of the metric g. Secondly, in deriving this problem, Dewitt utilizes language which is colored by an interpretive stance to which we might object. In particular, at a certain point in the following derivation it is assumed that our theory still contains a spacetime, though we do not have a fixed g as part of our background model and thereby have ceased to model a system that includes physical distances or durations of time. I will address this implicit interpretational stance and how it affects our understanding of the problem of time in §3.1.1.

In constructing LQG as well as canonical quantum gravity, we perform a "3+1 split" of the four-dimensional spacetime manifold represented by $\mathcal{M}$. The 3+1 spilt results in a one-dimensional time manifold $\mathbb{R}$ parameterizing a stack of three-dimensional spatial manifolds Σ, with metric γ_{ij}. This splitting of $\mathcal{M}$ requires that we split the four-dimensional Ricci scalar into an intrinsic and extrinsic part. The *extrinsic curvature* of our three-dimensional manifold from the perspective of a four-dimensional spacetime is given by the expression $K_{ij}K^{ij} - K^2$.[6] According to DeWitt, with this split, we can express the classical scalar constraint as:

$$C_3 = \gamma^{\frac{1}{2}}(K_{ij}K^{ij} - K^2 - R) \equiv H \tag{3}$$

Where R is the three dimensional Ricci scalar and encodes the intrinsic curvature of Σ. DeWitt reminds us that a generic Hamiltonian is written in terms of a kinetic energy

[6] K_{ij} is called the *second fundamental form* and K is the contraction of this form with the metric. Equation (3) below is a classical constraint equation. The corresponding quantum constraint is given by DeWitt (1967) as well as any number of books or review articles.

term minus a term for the potential energy. That the curvature of spacetime encodes the energy of the gravitational field according to GR, suggests that we interpret the scalar constraint as the sum of a 'kinetic-extrinsic" curvature energy term minus a "potential-intrinsic" curvature energy term (1967, p.1117). Given this interpretation, it is natural to interpret the scalar constraint as playing the role of the Hamiltonian in describing the system's evolution. It is for this reason that the scalar constraint is more commonly known as the Hamiltonian constraint (and labeled H).

After we upgrade to $\hat{H}$ and replace γ_{ij} with the state functionals $\Psi(\gamma_{ij})$, DeWitt concludes that for any time x^0 and state $\Psi(\gamma_{ij})$ which solves the Hamiltonian constraint $\hat{H}\Psi \equiv 0$:

$$\Psi^{\dagger}\hat{\gamma_{ij}}(x^0, \vec{x})\Psi = \Psi^{\dagger}\hat{\gamma_{ij}}(0, \vec{x})\Psi \tag{4}$$

Where $\hat{\gamma_{ij}}(x^0, \vec{x})$ is a field operator acting on the state Ψ. The only difference between the sides of this equation, is the value of the time variable x^0. DeWitt notes that all field operators, other than $\hat{\gamma_{ij}}(x^0, \vec{x})$, yield a similar result, and since the physics of quantum systems is encoded in the statistics produced by inner products of above kind, "the quantum theory can never yield anything but a static picture of the world" (1967, p.1119). In other words, no physical property, described by LQG, varies with time. This result is often named the "frozen formalism" or the "frozen dynamics" of LQG. The frozen dynamics is the first aspect of the problem of time and results from the Hamiltonian constraint. However, the usual response to the frozen formalism is to generate a second and distinct aspect of the problem by inferring that time is absent in LQG. What physicists usually mean by "time is absent" is that the coordinate time (x_0) does not model physical time. For instance, according to DeWitt:

> Instead of regarding this equation [the Hamiltonian constraint] as implying that the universe is static we shall interpret it as informing us that the coordinate labels x^{μ} are really irrelevant. Physical significance can be ascribed only to the intrinsic dynamics of the world, and for the description of this we need some kind of intrinsic coordinatization based either on the geometry or the contents of the universe. (1967, p.1120)

In other words, rather than interpreting the Hamiltonian constraint as requiring the dynamics of canonical quantum gravity to be frozen, DeWitt suggests that we re-evaluate our interpretation of x_0 as modeling time. Though I have used DeWitt's derivation of the problem and thereby am technically working with a different set of variables than that of LQG, the same issues and reasoning presented here are carried over to LQG. Throughout this paper when I refer to the problem of time, I am referring to the second aspect of the problem of time – that time "goes missing" in LQG. I will not use the alternative convention whereby 'the problem of time' refers merely to the frozen dynamics of LQG.

In summary, we begin with the formalism of GR which includes a differentiable manifold $\mathcal{M}$ and a psuedo-Rimannian metric, we quantize the gravitational field through either g or

$\mathcal{A}$, and do nothing to $\mathcal{M}$ besides split it into 3+1 submanifolds. Since our mathematical model does not include a metric, the physical world being modeled is assumed not to have the physical structures modeled by the metric. After replacing g with $\Psi(\gamma_{ij})$, we interpret the coordinate function x_0 on $\mathcal{M}$ as representing time and show that quantum expectation values do not change as we vary x_0. Finally, since we assume that physical geometry does in fact evolve with respect to physical time, we infer that x_0 must not model time after all. Since the Hamiltonian constraint requires that x_0 not model time, we say that time is missing in LQG and this is the problem of time. The reason why it is a "problem" that x_0 does not model time is because we have constructed LQG with the intention that it would, and have no agreed upon alternative for how to represent time in its place.[7]

This concludes my presentation of the problem of time in LQG. I will briefly argue that there is a *prima facie* conflict between this derivation of the problem of time and other commitments regarding the nature of "spin-networks" and spacetime in LQG. I will use this tension as a tool for gaining a better appreciation for why spacetime (and time) disappear(s) in LQG.

Central to the derivation of the problem of time is the commitment to treat $\mathcal{M}$ (without a metric) as representing a spacetime, and x_0 as representing time. And yet, in recent literature, one often hears claims to the effect that in LQG there is no spacetime fundamentally:

> The spin networks do not live in space; their structure generates space. And they are nothing but a structure of relations... (Smolin 2002, p.138)

> ...the quanta of the field cannot live in spacetime: they must build "spacetime" themselves... Physical space is a quantum superposition of spin networks...a spin network is not in space it is space. (Rovelli 2004, p.9,21)

> LQG thus seems to entail that space(time) is not fundamental, but emerges somehow from the discrete Planck-scale structure. (Wüthrich 2006, p.169)

> One influential idea based on so-called 'weave states' proposes that the spacetime structure emerges from appropriately benign, i.e. semi-classical, spin-networks. (Huggett, Wüthrich 2013, p.7)

Each of the locutions: 'Planck-scale structure,' 'spin-networks,' 'structure of relations,' etc., refer to the same thing. What a spin-network *is* will be unimportant for our purposes other than that it is whatever physical structure is represented by the states of LQG.[8]

[7] For an overview of possible solutions to the problem of time from the physics community see Isham (1991, 1992) and Kuchař (1992).

[8] For a discussion on the ontology of these networks see (Norton, in preparation).

According to the above quotes, these networks are interpreted as being a structure or a structure of relations which describe or perhaps *are* the Planck-scale structure of spacetime or, more accurately, "quantum spacetime."

Since the derivation of the problem of time assumes that spin-network states (Ψ) are dependent on time (x^0) and space ($\vec{x}$) axis, there seems to be a conflict between the derivation and interpretations of LQG which take spin-networks to be the pre-spatiotemporal "seeds" of spacetime. How is it that spin-network states are thought to be dependent on time and space if "[p]hysical space is a quantum superposition of spin networks"? (Rovelli 2004, p.21)

It turns out that there are ways of interpreting LQG which avoid this conflict: the idea is, we begin our analysis of LQG by assuming that spin-network states are dependent on space and time coordinates and then derive certain undesirable consequences, such as the frozen dynamics. We then use these consequences to motivate an interpretation of the states as representing a pre-spatio-temporal structure from which spacetime is built or emergent. In the following, I will represent four interpretations of LQG under which spacetime disappears fundamentally and for which spin-networks are described as being pre-spatiotemporal. The first interpretation is of the kind I mentioned above: the Hamiltonian constraint, through the resulting frozen dynamics, helps motivate a pre-spatiotemporal interpretation of spin-networks. However, there are interpretations of LQG which describe spin-networks as being pre-spatiotemporal for reasons independent of the dynamical considerations stemming from the Hamiltonian constraint. I will present three interpretations of this kind and use them to argue that, contrary to standard presentations, the Hamiltonian constraint is not the reason why time goes missing in LQG or, for that matter, the reason why spin-networks are thought to be pre-spatio-temporal. To this end, I will argue that the first interpretation, mentioned above, requires that we first adopt a notion of time which we, presumably, do not take to be the case.

3 Interpretations and the problem of time

In presenting the following interpretations of LQG, I do not mean to suggest that these are the only interpretations of the theory, and indeed they are not (see Norton, in preparation). I present the following interpretations in particular since they are present, either explicitly or implicitly present in the literature, or serve to highlight how different interpretations of 'spacetime' affect the nature of the problem of time in LQG. The first two interpretations are rather straight-forward whereas the last two are far less developed and more programmatic. I will do my best to present what is ontologically relevant from the latter two interpretations but will not provide a very complete account of them since it is unlikely that such an account is possible, at least in this stage of their development.

3.1 Rovellian

Though the following interpretation is based on the words and works of Carlo Rovelli, I do not claim that this is his interpretation. The following interpretation is Rovellian, if not Rovelli's. In the following, I will explain how, according to the Rovellian, spacetime disappears in LQG and the role which the Hamiltonian constraint plays in it doing so.

Taking a step back, in the context of classical general relativity, Rovelli interprets the diffeomorphism invariance of this theory to mean that the background manifold $\mathcal{M}$ is a gauge artifact of GR (2004, p.74). In fact, this is often how the manifold is treated by those wielding Einstein's hole argument (or more precisely, Earman and Norton's hole argument). Without reviewing a well-worn debate, what we are told to take away from the diffeomorphism invariance of GR is that, just because the theory utilizes $\mathcal{M}$ in modeling spacetime $\langle \mathcal{M}, g \rangle$, we ought not assume that there is a physically substantial manifold of spacetime points. Importantly, the diffeomorphism invariance of GR is found recapitulated in the theory of LQG and is the reason why the Rovellian interprets $\mathcal{M}$ as being mathematical gauge. The Rovellian does not begin her interpretation by tossing aside the manifold but rather concludes that it is gauge because of the diffeomorphism freedom of LQG which the Hamiltonian constraint plays a role in expressing.

As we have already discussed, the Hamiltonian constraint seems to require that our states be frozen in time and not to evolve as we move the state along the time axis x_0. Additionally, the diffeomorphism constraint is thought to require our states to be invariant under three-dimensional spatial diffeomorphisms (Gambini and Pullin 2011, p.93-94, Rovelli 2004, p.146, 225). Combining these constraints, we can interpret the four-dimensional diffeomorphism freedom of GR as having been projected along space and time dimensions in the context of LQG (Isham 1992, p.33). As a result, Rovelli concludes that the physics of LQG is independent of Ψ's relation to the manifold:[9]

> In fact $\mathcal{M}$ (the spacetime manifold) has no physical interpretation, it is just a mathematical device, a gauge artifact... There are not spacetime points at all. The Newtonian notions of space and time have disappeared... the spacetime coordinates $\vec{x}$ and t have no physical meaning...(2004, p.74)

> What Newton called "space," and Minkowski called "spacetime," is unmasked: it is noting but a dynamical object – the gravitational field... ...the gravitational field is the same entity as spacetime. (2004, p.9, 18)

[9] It is unclear how literally we should interpret Rovelli's repudiation of $\mathcal{M}$ as bearing any physical salience. It seems that at a minimum the global topology of "space" Σ has bearing on what our experiences of the world is like. For instance, if Σ is compact (i.e. has a dimension which is rolled up like a three-dimensional cylinder), then our theory of (quantum) spacetime should predict that we could travel a finite distance in one direction and come back to where we started.

Since all there is to spacetime is the metric field, according to Rovelli, and since the states of LQG represent a quantum version of the metric field, the states *ipso facto* represent a quantum version of spacetime itself! Consequently, there is no spacetime, *qua* classical structure, in LQG. Spacetime *qua* the physical structure represented by $\langle \mathcal{M}, g \rangle$, disappears according to the Rovellian, in two steps: the manifold is interpreted as having "no physical interpretation" (*ibid* p.74) and the metric field is replaced by quantum states.

Thus, for Rovellians, the Hamiltonian constraint helps motivate the "de-reification" of $\mathcal{M}$ and thereby the disappearance of spacetime in LQG. I say "helps motivate" since the Hamiltonian constraint does not do this alone. For the Rovellian, it is the combined effect of the Hamiltonian constraint, the three dimensional vector constraint, and the mathematics of the theory being independent of how we perform the 3+1 split (Thieman 2007, p.39), which together suggest that the manifold is a gauge artifact.[10]

Shortly, I will present three competing interpretations of LQG under which spacetime and time disappear and do so independently of the Hamiltonian constraint; however before doing so, I will argue that the Rovellian account of spacetime's disappearance requires that we adopt a rather counterintuitive notion of spacetime and time.

3.1.1 Conceptual problems with the derivation

Rovellians interpret $\mathcal{M}$ to be a gauge artifact in part because of the threat of the frozen formalism. The idea is that if we were to take the manifold "seriously" then the dynamics described by LQG would be frozen (as well as other oddities arising from the diffeomorphism constraint); thus, according to the Rovellian, the manifold must only be an artifact of our mathematics. The question which will drive the following discussion is "how 'seriously' must we take the manifold ($\mathcal{M}$) in order to derive the frozen formalism?"

Recall the following derivation of the frozen formalism and the resulting argument for the problem of time:

> We begin with the formalism of GR which includes a differentiable manifold $\mathcal{M}$ and a psuedo-Rimannian metric, we quantize the gravitational field through either g or $\mathcal{A}$, and do nothing to $\mathcal{M}$ besides split it into 3+1 submanifolds. Since our mathematical model does not include a metric, the physical world being modeled is assumed not to have the physical structures modeled by the metric. After replacing g with $\Psi(\gamma_{ij})$, we interpret the coordinate function x_0 on $\mathcal{M}$ as representing time and show that quantum expectation values do not change as we vary x_0. Finally, since we assume that physical geometry does in fact evolve with respect to physical time, we infer that x_0 must not model time after all. Since the Hamiltonian constraint requires that x_0 not model time, we say that time is missing in LQG and that this is the problem of time.

[10] In order to remove all traces of the manifold we also need to note that the geometric observables are "moduli"-invariant. See Rovelli (2004, p.267) or Norton (Part 1 of this dissertation).

Implicit in this derivation of the frozen formalism is the assumption that $\mathcal{M}$ still represents a spacetime and x_0 still represents time even though our model does not include a metric. What must spacetime and time be in order to still have a model for them though that model does not include a metric? In removing g from our model, it seems that we have at least two choices for what we take the geometric properties of a "spacetime" to be. We might think that a spacetime does not require any physical, metrical, or geometrical properties, and can be modeled by the bare manifold $\mathcal{M}$ alone. According to this suggestion, in removing g from our model, we do not thereby remove spacetime from the model. On the other hand, since we remove g in order to make room for the quantum geometry described by the states Ψ, we might think that spacetime, according to LQG, simply has quantum rather than classical geometric properties. Accoriding to this latter suggestion, LQG models spacetime *sans* g by representing its quantum geometry through Ψ. In either case, in order to derive the frozen formalism, we must interpret the Hamiltonian constraint as describing time evolution with respect to x_0, and this requires that a spacetime and time remain represented by the mathematics of LQG even though the theory does not include a classical metric.

We ought to object that it is infelicitous to think that there is a spacetime or time in either the bare-geometric world (represented by $\mathcal{M}$) or in the quantum-geometric world (represented by $\langle \mathcal{M}, \Psi(\mathcal{A}) \rangle$). Surely these structures are either too bare or too quantum to be models of spacetime (as opposed to quantum spacetime). In both the bare and quantum worlds, there is no well defined spatial distance between objects in "space" and no durations between moments of "time." Moreover, any physical structure which is defined in terms of or dependent on well-defined lengths or durations of time such as velocity, momentum, and force, will also be absent were the world without the physical geometric properties encoded by g.[11] Since so much of what we take a spacetime and time to be are missing under the bare and quantum descriptions, it is reasonable to suppose that a spacetime and time are not modeled by them. If this is the case, then spacetime and time disappear due to our interpretation of these concepts and not because of the Hamiltonian constraint.

Contrary to the above considerations, one might insist that either the bare or quantum world ($\mathcal{M}$ or $\langle \mathcal{M} | \Psi(\mathcal{A}) \rangle$) remains relevantly spatiotemporal to the extent that the Hamiltonian constraint entails the frozen formalism. However, what should we then infer from the frozen formalism? Should frozen dynamics signal, at this late stage, that time is missing when the lack of a physical metrical structure did not? I suspect, that for many, the answer is 'no.' And that, if time can survive the stripping of the physical metrical

[11] Since we explicitly quantize only the three-dimensional spatial projection of the gravitational field represented by the three-metric γ_{ij} or the three-dimensional gauge field $\mathcal{A}$, one might conclude that the *temporal* metrical structure has not been affected and that perhaps there are in fact durations of time in LQG. Though sensible, this conclusion is too quick. Since the formalism of LQG is built around an *arbitrary* splitting of the spacetime manifold into space and time submanifolds, whatever conclusions we infer about physical spatial lengths must also hold true for physical, temporal lengths. What we are calling 'space' and 'time' are artifacts of our mathematical representation which we can keep from affecting our interpretation of the physics by keeping in mind the arbitrariness of the 3+1 split.

structure then it can survive frozen dynamics. Consequently, if time does not survive in LQG it is not because of the frozen dynamics described by the Hamiltonian constraint. In the following, I will expand on this theme and introduce alternative interpretations for which spacetime and time disappear fundamentally in LQG and for reasons independent of the Hamiltonian constraint. The first interpretation is an explicit elaboration of some of the ideas just presented.

3.2 Composite substantivalism

The following interpretation is an example of one of a few interpretations which one might adopt regarding a spacetime's relation to $\langle \mathcal{M}, g \rangle$. In the context of LQG, since there is no physical structure having the properties described by g, what we take a spacetime's relation to $\langle \mathcal{M}, g \rangle$ to be, will make a difference as to whether or not there is spacetime in LQG.

According to composite substantivalism (CS), a spacetime has two components: a substantial basal structure represented by $\mathcal{M}$ and a physical geometry represented by g. According to CS, these two structures combine to form a spacetime which we represent by $\langle \mathcal{M}, g \rangle$. Since we replace g with the quantum variant $\Psi(\mathcal{A})$ in LQG, there is no spacetime. The physical structure of LQG is described by the ordered pair $\langle M, \Psi(\mathcal{A}) \rangle$ and this object, according to CS, does not have the physical properties required of a spacetime. I will stipulate that according to CS, $\mathcal{M}$ represents a substantival basal structure, $\Psi(\mathcal{A})$ represents a physical spin-network (or more accurately an "s-knot") responsible for quantum geometry, and together $\langle \mathcal{M}, \Psi(\mathcal{A}) \rangle$ represents a physically substantial *quantum* spacetime. Thus, while there is no spacetime in CS-LQG, there is a quantum spacetime. Importantly, time and spacetime disappear from CS-LQG because of our interpretation of 'spacetime' and not because of the Hamiltonian constraint.

3.3 Manifold quantization

In this section, I introduce what I am calling the "TaG" (topology and geometry) (and "trickle-down" interpretations. I will explicate these interpretations together since, unlike Rovellian or CS-LQG, these interpretations "quantize" the manifold $\mathcal{M}$. In just a moment, I will indicate, as best as possible, what "quantize" means in these contexts. However, as I mentioned at the start of this paper, these two interpretations are far more programmatic than well-developed versions of LQG; as such, I will not attempt to explain in very great detail how the manifold is "quantized" and will merely indicate what it might mean for spacetime and the problem of time if it were. In short, since TaG and trickle-down interpretations replace the manifold $\mathcal{M}$ with a quantum basal structure "$\hat{\mathcal{M}}$," in addition to quantizing the gravitational field, the physical structure described by these interpretations is that much less like a spacetime than were we to simply quantize the gravitational field.

According to TaG versions of LQG, the topology and geometry of classical spacetime

$\langle \mathcal{M}, g \rangle$ are explicitly replaced by some suitably quantized versions. The impetus behind TaG-LQG is a desire for a more radically background-independent theory of quantum gravity. How one goes about "quantizing" $\mathcal{M}$ however, is far from clear. In general, what 'quantization' means in this context is distinct from what it means when applying Dirac's quantization procedure. Moreover, as Isham (1991, p.137) notes, since $\mathcal{M}$ is a composite structure consisting of a set of points, topology, and differential structure, one has many options for which structures to quantize in quantizing $\mathcal{M}$ each of which corresponds to a different kind of TaG interpretation. For instance, according to Christopher Duston's version of TaG, we "quantize" $\mathcal{M}$ by encoding certain topological features of the manifold into the states Ψ by appending to the states an additional internal degree of freedom (2012, p.5). The states of Duston's extended-LQG are dependent on the manifold $\hat{\mathcal{M}}$ which does not have a well defined topology. Rather, these states encode $\hat{\mathcal{M}}$'s topology in addition to its geometry. Though both Isham and Duston have developed programs to quantize the topological structure of $\mathcal{M}$, one could instead attempt to quantize $\mathcal{M}$ through its differential structures or by *discretizing* the manifold's base set of points. In any case, however one goes about "quantizing" $\mathcal{M}$, in addition to g, the states of TaG-LQG represent physically distinct configurations of $\langle \hat{\mathcal{M}}, \hat{g} \rangle$: a quantum-spatio-temporal structure in all its manifold glories. To be clear, in replacing $\langle \mathcal{M}, g \rangle$ with $\langle \hat{\mathcal{M}}, \hat{g} \rangle$, we are not merely choosing to use some new mathematics, we are choosing to use a new mathematical model. No longer do we take there to be a physical structure represented by $\mathcal{M}$ and g. If we assume that a spacetime is only that physical structure modeled by $\langle \mathcal{M}, g \rangle$, then since we purposely peel away all the mathematical structure which we have deemed necessary for modeling a spacetime, there technically is no spacetime in TaG-LQG.

Though TaG theories have a modified mathematical structure from that of traditional LQG, they will, in general, incorporate many of its developments.[12] For instance, according to Duston's program, the physical states are simply modified spin-network states. Though TaG-LQG is less an interpretation than it is an extension of LQG, I have included it since spacetime disappears under it in a novel way: both structures which we used to model a spacetime in GR are removed from our physical model in LQG. This is different from the mathematical structure of Rovellian-LQG which still includes $\mathcal{M}$, if only as a mathematical artifact.

As for trickle-down interpretations, I intend for this interpretive-scheme to capture any and all interpretations for which the quantization of the gravitational field is thought to automatically affect a discretization of the base manifold, or something similarly destructive.[13] Unlike TaG interpretations which explicitly modify LQG to include a "quantization" of $\mathcal{M}$, according to trickle-down interpretations, $\mathcal{M}$ is indirectly "quantized" as an effect of having quantized the gravitational field. The trouble with trickle-down interpretations

[12] This term 'traditional' would be misleading if I did not add the caveat that there have been many attempts at formulating the theory of LQG and that no formulation should really be called traditional in a very serious sense.

[13] For a general discussion of this idea, see Isham and Butterfield (2001) and Isham (1991).

is that we are never told how quantizing the gravitational field affects $\mathcal{M}$. All we are told (more often implicitly than explicitly) is that the lumpy geometry of LQG (recall §1) somehow entails, or perhaps simply suggests, that the physical basal structure is not a continuum and thereby not modeled by $\mathcal{M}$. For instance, according to Butterfield and Isham:

> ... we mentioned the discrete spectra of the spatial area and spatial volume quantities: results that arguably suggest some type of underlying discrete structure of space itself. [Or, the] quantization of logically weaker structure such as differential or topological structure; these are called 'trickle-down effects'. (2001, p.78)

The idea, then, is that though we only explicitly quantize the gravitational field, the effect of doing so results in a discrete geometry which entails or suggests that the physical basal structure is more accurately modeled by something other than $\mathcal{M}$. For the time being, we do not need to understand why these interpretations take there to be trickle-down effects. The purpose in discussing these interpretations is to highlight their ontology: the physical basal structure is not modeled by $\mathcal{M}$. In order to show that trickle-down interpretations are more than a logical possibility, I will argue in the following section that something like trickle-down effects appear in Christian Wüthrich's and Karen Crowther's accounts of LQG. Additionally, I will present two arguments for why we might think that there are such effects. Before turning to these two tasks I will briefly discuss in which ways trickle-down effects require that we formally modify LQG and how some modifications threaten the technical fidelity of the resulting theory.

According to trickle-down interpretations, whatever basal structure there is to the world, this structure is not a physical continuum but rather something "logically weaker" (*ibid* p.78) which I will denote as $\hat{\mathcal{M}}$. As a result, spacetime disappears for the same reason it does in TaG-LQG: there is no physical structure having the form $\langle \mathcal{M}, g \rangle$. The trouble with both trickle-down and TaG interpretations is that we cannot simply replace $\mathcal{M}$ with $\hat{\mathcal{M}}$, without possibly, ruining the technical fidelity of LQG. Depending on which mathematical form $\hat{\mathcal{M}}$ takes, the result of exchanging $\mathcal{M}$ for $\hat{\mathcal{M}}$ might not result in a coherent formal structure. For instance, since the states of LQG are defined using holonomies along smooth curves in $\mathcal{M}$ (Norton, in preparation), if we replace the continuum with a discrete lattice, there will be no spin-network states. However, not all "replacements" are created equal: under Duston's program (2012), $\mathcal{M}$ is simply replaced with a continuum that lacks a well-defined topology ($\hat{\mathcal{M}}$) which Duston argues does not ruin the technical fidelity of the resulting theory (p.7). It is for this reason that Duston is able to utilize the formal results of LQG as well as its states. I note this potential complication since the extent to which TaG and trickle-down interpretations are more like extensions of LQG or new theories all together, depends on how much the "quantization" of $\mathcal{M}$ requires that we rework other aspects (e.g. the Hilbert spaces) of the theory.

3.3.1 Some particulars on trickle-down effects

In order to provide some flesh to the trickle-down interpretations and to show that they are not merely a logical possibility, I will briefly examine the views of Wüthrich and Crowther. Though neither author claims to endorse a trickle-down interpretation, both describe LQG in a way which suggests that there are trickle-down effects. In particular, both authors suggest that the discrete geometry of LQG somehow causes the basal structure of spacetime to be something other than a smooth continuum. In fact, the smooth manifold is described by these authors as being an emergent structure from some more fundamental discrete structure.

Starting with the physical states and predictions of LQG, according to Wüthrich, the process of taking the appropriate classical limit "should have as its effect the re-emergence of the continuous spacetime with its pseudo-Riemannian manifold" (2006, p.168, 174), and that the effect of following a certain mathematical procedure should "change the structure from discrete quantum states to smooth manifolds" (2014, p.24).[14] Consequently, according to Wüthirch, the fundamentally basal structure of LQG does not have the structure of a continuous manifold. While we are never told exactly why the continuous manifold is missing fundamentally, it's possible that the manifold being missing is somehow connected with the discrete geometry of LQG. Following his discussion of this discrete geometry (see §1), Wüthrich concludes:

> "The granularity of the spatial geometry – the 'polymer' geometry of space – follows from the discreteness of the spectra of the volume and the area operators... . Thus, the smooth space of the classical theory is supplanted by a discrete quantum structure displaying the granular nature of space at the Planck scale. Continuous space as we find it in classical theories such as GR and as it figures in our conceptions of the world is a merely emergent phenomenon." (2014, p.14)

If "continuous space" refers to the spatial continuum ($\mathcal{M}$), as is common, then according to Wüthrich, the discrete geometry of LQG somehow entails that the continuum is missing in LQG. However, if "continuous space" refers to a continuous spectrum of geometric areas and volumes, then this passage by Wüthrich does not suggest a connection between the discrete geometry of LQG and the supposedly missing manifold. The differences between these two senses of "continuous space" is subtle and important to keep clear. One might think that failing to have continuous geometric-spectra is the same as failing to have a continuous basal structure, but this is not true (see "Argument 1" below). Indeed, there are no formal constraints against defining a discrete geometry on a continuous manifold $\mathcal{M}$. In any case, though we are not told exactly why the smooth manifold is missing, since

[14] And more particularly, according to Wüthrich, the spacetime topology $\mathbb{R} \times \Sigma$ emerges or is required to emerge in LQG (2006, p.159-160).

it is missing according to Wüthrich, $\mathcal{M}$ – *qua* the smooth space of GR must not be a background feature of our mathematical model.

Similarly, throughout her account of spacetime emergence, Crowther claims that "... GR, and (continuum) spacetime..." (p.9) must be recovered and that LQG describes spacetime as a "cloud of lattices" (p.247). Taken at face value, these expressions suggest that the continuum of spacetime is affected in LQG; in particular, the second quote suggests that rather than having a continuous manifold of points in LQG, we have a cloud of lattices. Moreover, in noting that the geometric operators of LQG only increase in value as we increase the density of loops in a given region rather than becoming a better approximation of classical geometry (the details of this process are unimportant), Crowther notes:

> This is because macroscopic geometry is not recovered in the limit as the density of the weave (lattice) of loops goes to infinity. Intuitively, of course, it seems as though it would be the case that the continuum could be approximated in this way... The limiting procedure was thought to run analogously to that in conventional QFT, where a continuum theory is defined by taking the limit of a lattice theory, as the lattice spacing a goes to zero. (p.253-254)

In this quote, Crowther is primarily concerned with recovering macroscopic geometry, yet she expresses LQG's inability to do so in terms of a failure to approximate "the continuum." Again, the word 'continuum' is ambiguous on its own: standardly, it refers to the continuum ($\mathcal{M}$) used in modeling spacetime, but it could refer to the spectrum of possible area and volume values associated with a pseudo-Riemannian metric. Helpfully, Crowther's quote continues and contrasts this continuum against the lattice of lattice-QFT. The lattice of lattice-QFT is, of course, supposed to approximate and replace the smooth topological manifold $\mathcal{M}$. Indeed in this regard, Crowther later claims "...the limit in which the density of the weave states goes to infinity (or the "lattice spacing" goes to zero) fails to approximate continuum spacetime" (p.260). Thus, according to Crowther, the fundamental basal structure in LQG is something like a cloud of lattices: a discrete, non-continuous structure and thereby not modeled by $\mathcal{M}$ (p.247). In summary, according to both Crowther and Wüthrich, the continuum of spacetime is merely an emergent structure and is thereby missing fundamentally. Since there is no continuum, fundamentally, our mathematical model must include some non-continuous basal structure $\hat{\mathcal{M}}$ in place of $\mathcal{M}$. I have briefly described these aspects of Crowther's and Wüthrich's accounts of LQG in order to demonstrate that apparent appeals to trickle-down effects are sprinkled throughout the literature on LQG.

Neither Wüthrich nor Crowther explicitly endorse a trickle-down interpretation and thereby do not argue for the reality of trickle-down effects. Moreover, Isham and Butterfield merely allude that there may be such effects. Indeed, it is entirely possible that these authors are simply speaking metaphorically or employing a heuristic in talking about trickle-down effects. In whatever ways trickle-down effects have been employed in the past, we might want to know whether or not there are such effects. In the following, I will supply, on behalf of trickle-down interpretations, two arguments for the reality of such effects. One of these arguments is aimed at showing that quantizing the geometry of $\langle \mathcal{M}, g \rangle$ logically entails that the basal structure is itself discrete and not modeled by $\mathcal{M}$. The other argument is aimed to show that quantizing the geometry of $\langle \mathcal{M}, g \rangle$, in conjunction with something like Ockham's razor, requires that we modify our mathematical model from that of a continuous background to that of a logically simpler structure. I will argue that the first argument fails and will highlight challenges faced by the second.

Argument 1: Regions and surfaces on the continuum $\mathcal{M}$ can be infinitely divided into smaller and smaller sub-regions and sub-surfaces by taking smaller and smaller open sets (think about drawing circles on a piece of paper). However, as we have seen, physical regions and surfaces in LQG, cannot be infinitely parsed: there is a smallest size for any region or surface. Thus, if the physical basal structure were modeled by a continuum, as opposed to something like a lattice, there would be physical regions (those small circles) smaller than those allowed by the predictions of LQG. If this argument is sound, then the discrete geometry of LQG entails, along with other premises, that the physical basal structure of LQG is not modeled by a manifold on pain of contradiction.

In understanding where this argument goes wrong, it is helpful to recall the discussion from §1 where I was careful to distinguish between physical regions and surfaces on the one hand and physical areas and volumes on the other hand. The problem with Argument 1 is that the argument does not specify what it means by 'small' in reference to taking "smaller and smaller" open sets. Standardly, a small region in $\mathcal{M}$ is a region which has a small geometric measure, either a smaller area or volume. However, which definition of area and volume are we supposed to use in formulating the above argument? If area and volume are defined using the operators of LQG, then the first premise is false. It is not the case that regions can be parsed into smaller and smaller LQG-areas and LQG-volumes. However, if area and volume are defined in the first premise using a psuedo-Riemannian metric, then the argument equivocates since the second premise concerns LQG-areas and LQG-volumes and not pseudo-Riemannian areas and volumes. More importantly, our physical model does not include a metric; thus, while we are free to add a non-physical metric to $\mathcal{M}$, the Riemannian structures it defines (the circles we drew) are also non-physical. For these reasons, Argument 1 fails: either the first premise is false or the argument equivocates between two notions of 'small,' one of which is non-physical.[15]

Argument 2: Given that the geometric spectra are discrete, why postulate such a rich structure as the continuum? According to this argument, physical geometry could be

[15] One might wonder how a proper subset could not have a smaller volume than the set in which it is contained. This is possible in LQG due to the non-standard definition of area and volume utilized by the theory. See (Norton, in preparation) for more detail in this regard. In general, given a particular spin-network state, a proper subset might not defined a surface or region which is associated with any eigenvalue of area or volume. More generally though, every open set which is a proper subset of its closure has measure equal to and not less than its closure.

modeled with a simpler base set of points and topology and, as a result, the physical basal structure must actually have the logically simpler form $\hat{\mathcal{M}}$. There are two parts to this argument: first, that the physical geometry of LQG can be modeled with a simpler structure than $\mathcal{M}$, and second, that the physical world must not actually be a continuum. In this way, the continuum disappears fundamentally in LQG and does so by coupling the discrete geometry of LQG with Ockham's razor. The discrete geometry suggests that we don't need a continuum, and Ockham's razor cuts it out. There is nothing wrong with this argument though it does require that the physical geometry of LQG actually be captured using a simpler basal structure and that we take Ockham's razor seriously. Regarding the first condition, for it to be true that the physical geometry of LQG be captured by some logically simpler structure $\hat{\mathcal{M}}$, it must be the case that replacing $\mathcal{M}$ with $\hat{\mathcal{M}}$ does not modify the physical predictions of the theory. Thus, in order for Argument 2 to succeed, we must first demonstrate that the physical predictions of LQG are in fact invariant under this replacement (see §3.3).

To be clear, neither Isham nor Butterfield, nor Wüthrich, nor Crowther, employ either Argument 1 or 2. I have presented these arguments both as a guess towards what might be possible motivations for trickle-down effects and as a means of stimulating a more explicit conversation regarding their reality. If there are trickle-down effects in LQG, then spacetime and time are missing from the theory due to our interpretation of what is essential for spacetime: g and in particular $\mathcal{M}$, and not because of the Hamiltonian constraint.

4 Conclusion

Though spacetime and time might disappear in LQG, it is unlikely that the Hamiltonian constraint has much to do with this. According to the CS-LQG, spacetime disappears because the world does not include a physical metrical structure. According to both the trickle-down and TaG interpretations, spacetime disappears because there is neither a physical metrical structure nor a physical continuum in LQG.[16] According to these interpretations, spacetime disappears independently of the constraints and, in particular, time does not disappear due to dynamical considerations stemming from the Hamiltonian constraint. Contrary to these interpretations, according to the Rovellian, spacetime and time do disappear because of the constraints and, in particular, time disappears because of the frozen dynamics predicted by the Hamiltonian constraint. However, as we have seen, in order for time to disappear *because* of the Hamiltonian constraint, it is required that we endorse an odd notion of time: our notion of time must not require a metric and yet must require non-static dynamics. Unless this is what we take time to be, time does not disappear because of the Hamiltonian constraint but, presumably, because we removed the metrical structure from our model. Indeed, if time is essentially metrical, then the

[16] One does not have to be a substantivalist to think that the continuum represents physical information. The 'physical continuum' stands for whatever physical structure is represented by $\mathcal{M}$.

standard interpretation of the Hamiltonian constraint (as governing temporal dynamics), cannot even get off the ground. If the object of time requires a metric, then there literally is no time for the there to be a "Hamiltonian" constraint in LQG.

This paper is primarily concerned with how we might interpret the mathematics of LQG and the notion of spacetime in light of the Hamiltonian constraint and claims to the effect that spacetime disappears in LQG. By properly diagnosing the role which the Hamiltonian constraint plays or does not play in revealing the absence of time in LQG, we are better positioned to find a solution to the problem of time. In general, in order to find a solution to a problem it is helpful to know why there is a problem in the first place. Thus, so long as we continue thinking that time disappears because of the Hamiltonian constraint, we might look for time or variable dynamics in the wrong place. For example, it will not do to argue that some distinct sub-component of the Hamiltonian operator or some combination of operators, captures evolution with respect to x_0. As I have argued, there are independent reasons for thinking that x_0 is not the right structure upon which to model time: there is no metric, and it might be the case that there is no smooth manifold. Consequently, even if we were to find a structure which "evolves" states with respect to x_0, this would not entail that x_0 thereby models time or that the supposed "evolution" is a model of physical evolution.

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