Notes on Modal belief

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Nothing interesting is gained by placing complex statements as part of a belief, e.g. "You believe A and not B":

$$u : A \cdot \sim B$$

since you CANNOT infer from this either

$$u : A$$

or

$$u : \sim B$$

Such inferences require us to be perfectly rational which is not always the case. But interesting results do obtain by nesting complex statements into belief imperatives and so we will make statements of the form:

$$\underline{u} : A \cdot \sim B$$

$$\underline{u} : \Diamond(A \vee \sim A)$$

$$\Box\underline{u} : (x)(Fx \supset Fx)$$

A couple of rules/notes.

(1) We will be using other people as well (besides "I"), so we will have constructions which employ both $j : A$ (Josh believes A) and $\underline{j} : A$ (which is the command: "Josh believe A").

(2) The belief worlds of Josh might be different than those of Carl. Denote the different belief worlds of Josh as J_i and Carl as C_i where i=some number/index

(3) If you are told $W_3 j : A$ then you can infer two things: firstly, that W_3 is a belief world for Josh so you can convert the statement to some $J_i j : A$. We will call this rule WC (for world convert). Secondly you can infer that the object named by j must be a believing thing. So you can infer $\beta(j)$, which means that j is a believing thing. Call this rule BB (for Believing Being). Note, that if you are told $\beta(j)$, then you can use PE* (from the previous chapter) to infer $(\exists x)(x = j)$. Also, we will interpret statements like $\sim j : A$, which says that Josh does not believe A, to be committed to Josh's existence if true. (We do not need to take this stance, but in order to make things easier we will do so). This will allow us to infer $\beta(j)$ from $\sim j : A$.

(4) You can drop the box for almost any world, but can only drop the imperative $\underline{u}$: for belief worlds. The reason why you cannot drop the box for ANY world is because we want to make sure that when we have boxes which scope over belief statements that the box be dropped only into worlds which have the object in it. For instance take the statement $\Box j : A$ we don't want to drop the box into worlds in which Josh does not exists! If we decided to allow us to drop the box into any world, we would get that Josh exists in every world from statements like "necessarily Josh believes A." We would get this even when using the sophisticated system from last chapter. The whole point of introducing DU* was to prevent this from happening. When one says things like $\Box j : B$, what we mean is that for all worlds in which Josh exists he believes A. Yet from it we can prove $\Box(\exists x)(x = j)$! In other words it is a truth of logic that Josh exists necessarily if $\Box j : A$ and we don't want that. Here is the proof:

(Premise) $\Box j : A$

(1) $\sim [\Box(\exists x)(x = j)]$

(2) $\Diamond(x) \sim (x = j)$

(3) $W_1(x) \sim (x = j)$

(4) $W_1 j : A$ [using the unrestricted DB rule]

(5) βj [Using the new BB rule]

(6) $(\exists x)(x = j)$ [Using PE*]

(7) $W_1 \sim (j = j)$ [Using DU*(6,3)]

(8) $W_1 j = j$ [using SI]

(9) Contradiction so do RAA.

So before dropping a box which scopes over a belief statement, you have to check to see if the object said to be believing is a believing thing in that world. In other words: in order to drop $\Box((j : A) \supset B)$ into W_1 we need to know $W_1\beta(j)$. We will call this DB* and it requires you to cite two lines: the line with the box and the line with the Believing Being $W_1\beta j$

(5) Just to make things easier we will also make use of the Barcan Formulas $(x)\Box \equiv \Box(x)$ and $(\exists)\Diamond \equiv \Diamond(\exists)$. Cite this rule as BF.