WEAK DISCERNIBILITY AND RELATIONS BETWEEN QUANTA

JOSHUA NORTON

Abstract

Some authors (Muller and Saunders 2008, Huggett and Norton, 2013) have attempted to defend Leibniz's Identity of Indiscernibles through weak discernibility. The idea is that if there is a symmetric, non-reflexive physical relation which holds between two particles, then those particles cannot be identical. In this paper I focus only on Muller and Saunders' account and argue that the means by which they achieve weak discernibility is not through a quantum mechanical observable but an alternate mathematical construction which is both unorthodox and incomplete.[1] Muller and Saunders build a map from numbers to a set of observables (acting on the singlet state) and out of this map construct a weakly discerning formal relation. What Muller and Saunders' do not provide is a worked out account of how such maps pick out physical relations between particles. Though I argue that the method pursued by Muller and Saunders ultimately fails, I provide reasons for thinking that the PII might not actually be threatened by entangled states.

[1]It is unorthodox since it is not a construction used in quantum mechanics to specify physical properties, and it is incomplete since the general account for how such constructions model properties of quantum systems is not provided.

1. Introduction

In their paper, "Discerning Fermions," Muller and Saunders (2008) argue that identical particles are weakly discerned by having opposite spin. This is in response to the long-standing concern that identical particles, like fermions, violate Leibniz's Principle of the Identity of Indiscernibles (PII). However, if "identical" particles are discernible by their spins then they do not differ merely *numerically* and thus do not violate the PII. If Muller and Saunders are correct, then they will have successfully demonstrated that one form of the PII is immune from challenges posed by identical particles. The first half of this paper will involve laying out the relevant issues as well as Muller and Saunders' position. In the second half of this paper, I will argue that Muller and Saunders' account fails since they don't make use of quantum observables and what they do make use of, we are not justified in interpreting as a physical relation.[2] Without a physically pertinent relation, one cannot even begin the process of weak discernibility. Finally, I argue that we have good reasons for being suspicious that there are two particles which are in need of being discerned. Without there being two things, the PII is not threatened by quantum mechanics.

2. The Challenge from Identical Particles and Weak Discernibility

Assuming some familiarity with both the PII and with the challenge raised by identical particles, I will be brief in my retelling of this story. Leibniz posited seven principles which he largely took as being self evident, the Principle of the Identity of Indiscernibles being one of them. This principle states that if two objects are indiscernible then they are identical or, a slightly more interesting

[2]Both Huggett and Norton (2013) as well as French and Redhead (1988), make a similar mistake in how they build physical relations. Huggett and Norton make the same assumptions as Muller and Saunders.' French and Redhead consider relations built out of conditional probabilities of single particle operators and provide no argument that these relations represent physical relations of the particles themselves. See Huggett and Norton (2013) for remarks on French and Redhead's use of non-symmetrized observables.

formulation, "That it is not true that two substances may be exactly alike and different merely numerically, *solo numero*" (Leibniz, 1686). I say more interesting because this quote makes clear that if there are two numerically distinct things then they must be discernible; however, the quote also claims that if two things are indiscernible then they must not be even numerically distinct: there must be only one thing. The second conditional will play an important role at end of this paper. Leibniz's principle can be written in the following logically equivalent ways. The first most faithfully captures the words of Leibniz, but the third is the most common way of thinking of the principle:

(1) $$\neg[\exists x \exists y[(x \neq y) \cdot \forall F \in \{F\}|(F(x) \equiv F(y))]]$$

(2) $$\forall x \forall y[(x \neq y) \supset \exists F \in \{F\}|\neg(F(x) \equiv F(y))]$$

(3) $$\forall x \forall y[\forall F \in \{F\}|(F(x) \equiv F(y)) \supset (x = y)]^3$$

[3]Some authors include both monadic and relational properties in the set F; however, Leibniz's view on relations suggest that we keep these cases separate. Leibniz scholars debate whether Leibniz was simply an antirealist regarding relations, in which case relations would certainly not be included in his PII, or whether relations, for Leibniz, are unique in the sense that no two objects could partake of the same relation as some other distinct pair of objects. For instance, the relation of brotherhood shared between two men would not the same as the relation of brotherhood shared between any other distinct pair of men. Consequently, if a relation is satisfied by a pair of distinct objects, the relation fails to be reflexive and as a consequence weak discernibility is easy to satisfy. Under this second interpretation it again seems unlikely that relations were originally included in the PII. For a discussion on Leibniz' views see Mugnai (1992).

Where {F} represents the set of monadic properties for the objects in our domain. According to this formulation, we define that two objects, a and b, are *strongly* discerned if, and only if:

$$\exists F \in \{F\} | \{[F(a) \land \neg F(b)] \lor [\neg F(a) \land F(b)]\} \tag{4}$$

If strong discernibility (4) is met, then the inference in (3) cannot go through. In such a case, one could then use Leibniz's Principle of the Indiscernibility of Identicals to infer that the "substances" must not be identical. However, in the quantum case, it seems that (4) fails and so the inference in (3) does go through.

The literature recognizes other means of discerning objects including that used by Muller and Saunders: two objects, a and b, are defined to be *weakly* discerned if, and only if, there is a physically relevant dyadic predicate that is symmetric and non-reflexive[4] when applied to the two objects:

$$\exists F(a, b) | [F(a, b) \land F(b, a) \land \neg F(a, a) \land \neg F(b, b)]^5 \tag{5}$$

The idea behind weak discernibility is that if two objects are identical then $F(a, b)$ is equivalent to $F(a, a)$ and the above condition fails; thus, if it does hold then a and b cannot be identical. Though the inclusion of weak discernibility shifts the debate away from the exact form of Leibniz's principle, the spirit of the debate remains very much the same.

[4]Quine (1976) as well as Muller and Saunders (2008), formulate the principle of weak discernibility in terms of irreflexive relations and not non-reflexive relations. However, I see no reason to require the logically stronger relation in the context of discerning identical particles.

[5]Muller and Saunders (2008, 528-529). The authors also note that weak discernibility can be satisfied by a relation which is reflexive and symmetric and which does not hold between all non-identical objects. Examples of such relations are 'identical to', or perhaps 'same haecceity.' Such relations will not be of much use to us.

Quantum mechanics challenges the truth of Leibniz's principle in that identical particles in states like the singlet state:

$$\Psi = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) \tag{6}$$

are thought to be distinct particles and yet have no property to discern them. The term 'identical particles' is unfortunate given the current context. To be clear in the context of quantum mechanics, all that is entailed by two or more particles being identical is that each has the same value for their non-dynamical properties such as their mass and charge. Identical particles are not necessarily identical in a logical or ontological sense. For the sake of sticking as closely to the terms of Leibniz's project I will be assuming that we have two objects which do differ at least *numerically*, but there is an important sense in which entangled states may deny this and thereby trivialize the question of discernibility.[6]

The singlet state in (6) describes an entangled two particle system expressed in terms of $\hat{z}$-spin, while the particles' location has been suppressed. In order to be relevant for the PII, we insist that the implicit portion of the wave function be identical and that the particles follow the same path through space (this is why it has been suppressed). This state is physically realizable as two entangled electrons in the same orbital of some atom. In such a situation, for all the properties that are specified by quantum mechanics, the particles share said properties. In other words, there is no property held by one of the particles and not by the other, yet we do not think that these

[6]Given the holistic nature of entangled states, one might question whether or not it even makes sense to speak of having two particles when the state is entangled. It is plausible that particles lose their identity through entanglement and become *only* a new unity. If this is correct, then the PII cannot be challenged by entanglement since there are no longer two things which differ *solo numero*. I will elaborate on this point towards the end of the paper.

particles are ontologically one–as would be required by the PII. Consequently, many have taken the existence of such particles as being counterexamples to the PII.

Though identical particles fail to be strongly discernible, Muller and Saunders claim that they are weakly discerned by the relation 'has opposite spin.' This relation is symmetric and irreflexive and yet in order to justify its application to the singlet state we need an 'opposite spin'-observable which when applied to the singlet state yields an appropriate eigenvalue. A common assumption, though far from obvious, is that there is a one-to-one mapping from physical properties to quantum mechanical hermitian operators (observables), such that if a state Ψ is the ith eigenvector of the observable with eigenvalue a_i then the state is said to have the property associated with this eigenvalue. This association between properties and eigenvalues is often referred to as the "eigenvector, eigenvalue link" (EE-link) and serves to coordinate hermitian operators with properties of a system. In order for us to have a mathematical framework complete enough to describe quantum phenomena, we require that our mathematical machinery be able to represent every physical property that might be possessed by our system. Thus, if we want to play the game of modeling quantum mechanics at all, we must assume that the aforementioned mapping is at least injective.[7]

[7]I am leaving open the question of surjectivity; in particular, if there are super selection rules then the mapping will not be surjective. Relatedly, one might worry that injectivity also fails. It seems that there are quantum facts which we do not represent with Hermitian operators. Such facts might include selection rules, super selection rules, the Stone-von Neumann theorem, or that all systems are represented as vectors in a Hilbert space. Two comments on this: (1) these properties are not properties of physical systems, at best they are relations which hold between the physical world and some language(s) which we use to model the world. (2) The ongoing assumption in literature is that meeting the EE-link is the gold standard for predicating of quantum systems and that projects which diverge from it, ought to be held as suspect. Setting these concerns aside, I more than happy to consider alternate mathematical devices for modeling properties of our quantum systems; however I will not do so here since the EE-link is something which Muller and Saunders are going to want to abide by since they aim to present an orthodox defense of the PII.

Muller and Saunders (533) use the following generalized operator to build the relation of having or failing to have opposite spin:

$$\sum_{i,j=1}^{d} P_{ij}^{(a)} P_{ij}^{(b)} \tag{7}$$

Without getting into too many of the details, the superscripts on the single particle operators $P_{ij}^{(a)}$ pick out which slot of the tensor product the operators act and the subscripts refer to vectors of a given eigen basis. For example, $P_{\uparrow\downarrow}^{(2)} = \mathbb{I} \otimes (P_{\uparrow} - P_{\downarrow}) \otimes \mathbb{I} \otimes ...$, where the $P_{\uparrow}$ projects onto the $\uparrow$ $\hat{z}$-spin eigen vector and the and $P_{\downarrow}$ onto the $\downarrow$. Muller and Saunders (533, 535) define the following two relations and claim that both are able to weakly discern the identical particles:

$$Z_{-2}(a,b) \quad \text{iff} \quad \sum_{i,j=1}^{d} P_{ij}^{(a)} P_{ij}^{(b)} \Psi = -2\Psi \tag{8}$$

$$Z_{2}(a,b) \quad \text{iff} \quad \sum_{i,j=1}^{d} P_{ij}^{(a)} P_{ij}^{(b)} \Psi = 2\Psi \tag{9}$$

In the case considered by Muller and Saunders, the $P_{ij}^{(\cdot)}$ operators each reduce to the Pauli spin matrix σ_z and we get the following refinement:

$$Z_{-2}(a,b) \quad \text{iff} \quad 2\sigma_z^a \sigma_z^b \Psi = -2\Psi \tag{10}$$

$$Z_{2}(a,b) \quad \text{iff} \quad 2\sigma_z^a \sigma_z^b \Psi = 2\Psi \tag{11}$$

For a=1, b=2 ($a \leftrightarrow b$) the relation holds:

$$Z_{-2}(1,2) \equiv 2\sigma_z \otimes \sigma_z \Psi = -2\Psi \tag{12}$$

For a=b=1 the relation does not hold:

$$Z_{-2}(1,1) \equiv 2(\sigma_z)^2 \otimes \mathbb{I}\Psi \neq -2\Psi \tag{13}$$

For a=b=2 the relation does not hold:

$$Z_{-2}(2,2) \equiv 2\mathbb{I} \otimes (\sigma_z)^2\Psi \neq -2\Psi \tag{14}$$

In other words Z_{-2} holds iff $a{\neq}b$. The relation Z_2 holds under the exact opposite conditions. Thus, the particles are weakly discerned by the relation Z_{-2} and the PII is immune from challenges posed by identical particles. For a more detailed account see Muller and Sauders (2008).

Although Muller and Saunders' account is straight forward it is far from unproblematic. Equation (10) captures three different equations each with its own observable. As we select particular values for a and b we construct new mathematical objects, see (15). Consequently, the formal relation Z_{-2}, used to weakly discern, is not a Hermitian operator on the system and we cannot apply the EE-link to it. Since the EE-link is not applicable to Z_{-2} we have no reason for thinking that it represents a physical relation. In the following section I will develop a series of criticisms which stem from this fact.

3. Challenges

Muller and Saunders are aware that traditionally the EE-link (roughly what they refer to as StrPP)[8] is used to pick out monadic predicates, they none-the-less insist that it can also pick out polyadic predicates:

> Finally, a note on relations. When physical system S is (taken as) a composite system, built up of other physical systems, some of the properties of S determine and are determined by relations of its constituents. ... Consequently, both WkPP and StrPP [EE-link], although giving rise to properties of S and of its subsystems (expressed by monadic predicates), equally provide conditions for the ascription of relations among constituents of S; the magnitude A may itself be relational (as in relative distance), and likewise the operator A corresponding to it. (This is why one does not need to introduce relation postulates in addition to property postulates.) Typically, however, as we shall see in the next section, where authors have made use of WkPP or StrPP [EE-link], they have used them only to consider monadic properties – that is to say, from our point of view, they have not made use of either property postulate to ascribe relations among constituents of S which is the key step that we shall be taking in this paper.
>
> (Muller and Saunders, 515)

According to Muller and Saunders, the following are the conditions for ascribing a physical relation to a formal relation as in (10):

[8]According to Muller and Saunders "We represent a quantitative physical property mathematically by the ordered pair $\langle A, a\rangle$ where A is the operator which corresponds to physical magnitude $\mathcal{A}$ and $a \in \mathbb{C}$ is its value. According to the Strong Property Postulate (StrPP), a physical system S having state operator $W \in \mathcal{S}(\mathcal{H})$ possesses quantitative physical property $\langle A, a\rangle \in \mathcal{MS}(\mathcal{H}) \times \mathbf{C}$ iff W is an eigenstate of A that belongs to eigenvalue a" (513).

> When the projectors under consideration belong to the spectral family of magnitude-operator A, assumed to be physically meaningful, they are themselves physically meaningful; by the WkPP, when the system is in the state W, so is relation Rt, which is defined in terms of them (Req1).
>
> (Muller and Saunders, 532)

Where the condition (Req1) is "all properties and relations should be transparently defined in terms of physical states and operators that correspond to physical magnitudes, as in WkPP, in order for the properties and relations to be physically meaningful" (Muller and Saunders, 527). Here Muller and Saunders's use the WkPP which is simply a weaker version of the StrPP (Muller and Saunders, 514) and will make no difference to my argument. In summary, according to Muller and Saunders, what justifies assigning a physical relation, 'opposite spin', to Z_{-2} is merely that such a relation gains physical transparency by piggy-backing off the physical transparency of the observables which show up in (10). Before examining this claim I want to make clear that I am in no way questioning whether the observables which show up in (10) have physical meaning and am only questioning how physical content is forced onto a formal relation built out of them.

Firstly, Z_{-2} is not an observable itself, but is map from $\{1,2\} \oplus \{1,2\}$ to the following set of observables (acting on Ψ):

$$\{2\sigma_z \otimes \sigma_z \Psi, \;\; 2(\sigma_z)^2 \otimes \mathbb{I}\Psi, \;\; 2\mathbb{I} \otimes (\sigma_z)^2 \Psi\}^9 \tag{15}$$

Z_{-2} is associated with three different observables, each of which seem to be "manifestly" physical and yet each of which could bear a distinct physical meaning. What then is the relationship between this set of observables and Z_{-2} whereby the map becomes associated with a single physical relation?

[9]As discussed by Huggett and Norton (2013), we should be cautious of these observables since they are multiples of the identity on the space of fermions and are not symmetrized.

There are three questions: (1) why must the map represent any physical property of the system or of the particles which make up the system, (2) why is this map associated with a dyadic property (opposite spin) and not a property of some different arity, and (3) even if the mapping is associated with a dyadic property, why must we think that this relation is symmetric and irreflexive? Surely the *formal* relation $Z_{-2}(a, b)$ is symmetric and irreflexive on slot indices a and b, but why must the observables in (10) bind together to produce a *physical* relation which is symmetric and irreflexive on the particles? The criterion we are in search for needs to be more than that the objects mapped onto are physically meaningful, or are individually a relation of a certain kind since in general it is false that maps have the same properties as the objects in their image.

What Muller and Saunders need to supply is an EE-link for relations which identifies when n-valued functions onto sets of observables pick out m-ary properties. Such a link would unpack what it means for a relation to be "transparently defined" by selecting which formally defined relations represent physical properties and what the relevant physical property is. However, an EE-link for relations is neither provided for by Muller and Saunders nor is it part of standard quantum mechanics.

As things stand, we cannot do much with Muller and Saunders' project until we are given more reason for thinking that Z_{-2} represents something physical. However, for the sake of argument let us assume that Z_{-2} is physically meaningful and consider questions (2) and (3). If Z_{-2} does represent something physical, are there reasons for associating it with the relation 'opposite spin?' Not clearly. Analyzing (12)-(14) and how the operators act on the singlet state does not leave one with any assurance that any weakly discerning relation is being described by them. For instance, there is nothing ruling out the possibility equation (12) tells us that the state has total spin equal to zero while (13) and (14) tell us that the total spin fails to be either greater or less than zero. If these are the proper interpretations suggested by the individual equations (12), (13) and (14) then it would be hard to argue that Z_{-2} represents a relation since the individual observables are monadic properties of the system. Similarly, in order for Z_{-2} to represent a weakly discerning relation on the particles, equations (12)-(14) must not bind together to suggest some three place relation between the particles and their environment or *etcetera*. Somehow we must use the information in (12)-(14) to rule out all contrary interpretations and leave only an interpretation attached to some weakly discerning relation. Without this relation, Muller and Saunders have only a physically salient map (since we have granted them this) and no means to weakly discern. That equations (12)-(14) do select such a relation is neither argued for by Muller and Saunders nor is it even clear what such an argument would look like.

Before exploring other motivations for why we ought to think that 'opposite spin' is true of the singlet state, I just want to highlight one further worry about Z_{-2}. If Muller and Saunders are correct in their claim that $Z_{-2}(x, y)$ represents a physical relation since it is built out of physically salient Hermitian operators then it must also be be true that $Z(x) \equiv Z_{-2}(x, a)$ represents a physically relevant monadic property since it too is also *built out of* physically salient Hermitian operators.[10] Moreover, since it is not the case that both $x = 1$ and $x = 2$ satisfy $Z(x)$, the particles are strongly discerned by it. However, we had already concluded that the particles are not strongly discerned.

We are able to by-pass our previous conclusion by utilizing non-Hermitian operators to represent monadic properties of the particles. If we think that we have made a mistake in getting to this conclusion (and we have), then I suggest that we have also made a mistake in the case of weak discernibility. Perhaps in filling out or adding to Muller and Sauders' conditions for why $Z_{-2}(x, y)$ represents a physical relation we will discover conditions which fail when applied to $Z_{-2}(x, a)$.

[10]Where a is either 1 or 2.

However, until these conditions are given we should have no confidence that $Z_{-2}(x, y)$ is any *more* physical than $Z_{-2}(x, a)$.

So far we have not found anything in the formal structure of Z_{-2} acting on Ψ which suggests 'opposite spin' or any other symmetric and irreflexive physical relation. Why then think that it does? With respect to their claim that $Z_{-2}(a, b)$ represents 'opposite spin,' Muller and Saunders say:

> Relation Z_{-2} is the one in footnote 5 of (Saunders [2003a], p. 294): 'has opposite direction of each component of spin to.'(Muller and Saunders, 535)

In this footnote Saunders argues:

> The most general anitsymmetrized 2-particle state is $\Psi = \frac{1}{\sqrt{2}}(\phi \otimes \psi - \psi \otimes \phi)$ where ϕ and ψ are orthogonal. Analogues of operators for components of spin can be defined as $\mathbf{S} = P_\phi - P_\psi$, where P_ϕ, P_ψ are projections on the states ϕ, ψ. Each of the two particles in the state Ψ, has opposite value of $\mathbf{S}$. But no particle can have opposite value of $\mathbf{S}$ to itself.[11]
>
> (Saunders, 2003a)

What does the heavy lifting in predicating opposite **S**-value (opposite spin) to the particles is apparently Saunders' assertion that each of the two particles in the state Ψ happen to have opposite values of spin. This is surely troubling; if we are not going to reason in a circle we must have some way of verifying that the particles do in fact give opposite values of **S**. Yet given that the state is

[11]There are formal problems with Saunders' **S**: it is a single particle observable and cannot be applied to the singlet state. This is corrected in Muller and Saunders; though, their observable is not symmetrized. In this same footnote, presumably in defense of his position, Saunders quotes Mermin (1998). In this paper Mermin gives a non-standard interpretation of quantum mechanics under which all there is, are relations and no relata. According to this interpretation there are no particles to weakly discern since there are no particles. The role which Mermin's interpretation is supposed to play in Saunders' account is far from clear. Surely such an interpretation cannot help decide the fate of the PII which assumes that there is relata in the world.

entangled we have no way of isolating any single particle to determine its spin. Saunders claims that the observable **S** does pick out opposite eigenvalues when applied to the particles in the singlet state; however, in order to make this claim one would need to first identify which part of the singlet state, represents some single particle. Though we know that such talk is nonsense, perhaps if we were to look at the actual terms of the $\hat{z}$-spin-expansion of the singlet state, we might be able to find some motivation for thinking that 'opposite spin' has something to do with the singlet state. It sure seems like the particles in (16) have opposite spin.

$$\Psi = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) \tag{16}$$

How does one dissect this state to apprehend one particle at a time and thus come to believe that each particle has the opposite **S**-value from the other? If one were to assume that the slots of each term of the singlet state represent the individual particles, then we might conclude that the particles have opposite spin. For instance, let:

$$\text{Particle 1} \quad (\uparrow \otimes \cdot) \quad \text{as well as} \quad (\downarrow \otimes \cdot) \tag{17}$$

$$\text{Particle 2} \quad (\cdot \otimes \downarrow) \quad \text{as well as} \quad (\cdot \otimes \uparrow) \tag{18}$$

Then according to our assumption, the first slot in $\uparrow\downarrow$ represents the first particle and its properties and the second the second particle and its properties, and similarly for $\downarrow\uparrow$. Since the first particle in $\uparrow\downarrow$ has the opposite value of spin as the second particle, we conclude that the particles have opposite spin. Not forgetting that there are two terms, we check the second term $\downarrow\uparrow$ and note that the particles represented here also have opposite spin. Therefore by the linearity of (16), it is true of the singlet state that the particles have opposite spin.

The mistake of course is that $\uparrow\downarrow$ does not represent anything physical: no identical particles are represented by $\uparrow\downarrow$. The eigenvector $\uparrow\downarrow$ is no more essential to the physical description of the state than the minus sign connecting the other nonphysical pieces of syntax. As a whole, the state has physical content but none of its parts do. To treat the terms $\uparrow\downarrow$ or $\downarrow\uparrow$ as individually giving us physical information of the state is to treat the singlet state as akin to a statistical mixture rather than a genuine entanglement.[12] This line of reasoning corrects an assumption we may have had regarding the EE-link: we may have assumed that since observables are linear in the wave function that the properties associated with them are too. This of course is false. Physical properties only hold on physical states. The EE-link associates properties with physical states and not with non-physical terms of a state's decomposition. Moreover, implicit in my treatment is a denial that slot indices always represent single particles. Rather, only those slots of factorized states, represent single particles: slots on non-physical syntaxt cannot represent physical particles.

In summary of these arguments: the singlet state is 'Ψ' and focussing too much on the expansion '$(\uparrow\downarrow - \downarrow\uparrow)$' may tempt us into thinking that its pattern of arrows is revealing hidden relations between the single particles. We must conclude that there is nothing in the singlet state's description that warrants attributing opposite spin to its particles. Moreover, since, as we have seen, there is nothing in the formal structure of Z_{-2} which warrants our association of it to the relation 'opposite

[12]Let us lay this objection aside and assume that it makes sense to read physical properties off non-physical syntaxt, we still need a story as to why 'opposite spin' is selected and not 'definite and opposite spin?' Surely it is true of both $\uparrow\downarrow$ and $\downarrow\uparrow$ that the particles have definite and opposite spin, and thus should we not also say of the singlet state that its particles have definite and opposite spin? Yet this is false: the particles described by the singlet state do not have definite spin and may or may not have opposite spin (this is the contentious claim we are exploring.) This example highlights the danger inherent in inferring state properties from non-physical states.

spin,' quantum mechanics simply does not provide us with a description of the particles' spins.[13] If quantum mechanics does not provide a description then why do we think there is one?

4. "*solo numero*"?

In the following I will argue that anyone wishing to defend the PII by using properties like 'opposite spin' have good reason for not attempting it. In particular, predicates like 'opposite spin' are only true if there are two things represented by the singlet state and this is the exact situation required for the PII to come under attack. The PII is not threatened by the singlet state if it does not represent two things which are numerically distinct. In the following I will introduce considerations for interpreting the singlet state as representing only an emergent whole. (Of course many others have also considered this topic and I claim very little originality in pursuing it.)

Firstly, because of their entanglement we know that the singlet state cannot be written as a factored state. When the state is factored we can use the slots of the tensor product to refer to the single particles. When the state is entangled we can no longer refer to particles this way since there are no factored slots. For this reason, though it might be true that there are two particles represented by the singlet state it is not true that the singlet state refers to them (at least as we normally refer to single particles in joint states). One might object that though the singlet state does not *directly* refer or name two things, that it does refer *indirectly* since it is built out of a symmetrized sum of product states which do refer to two things. This objection suffers from the same problem which we have already encountered: the terms of the singlet state have no physical interpretation in the context we are considering. In the case of identical particle systems the product

[13]It might be the case that Z_{-2} ought to be associated with 'opposite spin;' however, given our current understanding of quantum mechanics, we are not warranted in making this association. In order to be warranted, we require a more robust EE-link than the one we currently have.

states which linearly compose to form the singlet state do not themselves refer to anything let along individual particles.

If the singlet state does not refer to the two particles composing the system what does it refer to? It refers to the system as a whole. Indeed, the quantum effect of entanglement entails that the system referred to by the singlet state strictly contains more information than the information of the subsystems which went into it.[14] We can decompose the density operator associated with the singlet state, into mixed density operators on its subsystems. Since the decomposition leads to a mixture, it does not uniquely recompose into the composite singlet state.[15] We can of course use the reduced density matrices to describe the subsystems, but we cannot use them to describe the state as a whole. In order to describe the state as a whole, we need more information than can be obtained from the descriptions of the component subsystems. Consequently, not only does the singlet state refer to the whole system but it refers to an emergent whole.

Why then do we think that there are two particles described by the singlet state? What seems to be the right response is to note that the singlet state lives in the tensor product of two single particle Hilbert spaces. Surely all such states represent two things? The trouble is that Hilbert spaces do not come with built in interpretations. It is we, the users of the theory, which get to name and interpret our Hilbert spaces (within reason) as part of our model building. For instance, we are free to adopt the "received interpretation," and interpret the tensor product of two single particle Hilbert spaces as representing the space of two things or we can choose to have a nuanced interpretation of such spaces. Contrary to the received interpretation, perhaps only factorizable states represent more than one thing, while all non-factorizable states represent only single unities? Indeed, no experiment will decide between the different *metaphysics* assumed in these two interpretations.

[14]Hughes (1992, 150-151, 250).
[15]Nielsen and Chuang (2000, 106).

Thus, whether or not factorizable states in "two particle Hilbert spaces" actually represent two things is not decided for us by quantum mechanics.

The following dialectic is not so much a defense of the PII as much as it is a clarification of what's at stake. If we endorse the received interpretation, the singlet state will be interpreted as somehow describing two particles even though the state also describes an emergent "unity" and provides us no purchase in using the state formalism to refer to two things. Moreover, by endorsing the received interpretation we come into direct conflict with the strong form of the PII. The PII is threatened by the singlet state only if there are two things being described by it. Thus, there is a sort of intuition "tug of war" between the PII and our interpretation of tensored Hilbert spaces.

On the one side we have the PII and the fact that entangled states do not explicitly refer to two things and on the other side we have the received interpretation of tensored Hilbert spaces. If the PII is true then the singlet state does not refer to two numerically distinct particles and the received interpretation is false. However, if the received interpretation is correct then there are two numerically distinct particles and the strong form of the PII is false. Thus, the true challenge of the PII from quantum mechanics rests, rather weakly, on our interpretation of tensored Hilbert spaces. Moreover, if one wants to defended the PII from quantum theory one need only to adopt the nuanced interpretation; however, by doing we make it so that all relations, like Muller and Saunders' 'opposite spin' are ill posed when applied to states like the singlet state. Thus, Muller and Saunders have good reason for not predicating 'opposite spin' of the singlet state and thereby can avail themselves of the nuanced interpretation which effectively cuts off the PII from attack.

In summary, I have provided three challenges to predicating 'opposite spin' to states like the singlet state. The first challenge stems from Muller and Saunders' emphasis that their $formal$ relation Z_{-2} should be identified with a *physical* relation. I have argued that there is nothing in the formal structure of Z_{-2} for thinking that it represents any physical property of the state, nor that it is a physical relation, nor that the physical relation is suitably structured to weakly discern. Secondly, I have noted that there is nothing in the singlet state itself to warrant thinking that the particles contained therein have opposite spin. Though I have argued that Muller and Saunders' defense of the PII does not work, I have also noted that the PII is threatened by quantum mechanics only rather weakly through a particular interpretation of tensored Hilbert spaces. Moreover, I have argued that anyone who would defend the PII with weakly discerning relations is better off not attempting to do so since the interpretation it requires is what opens the PII up to attack to begin with.

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